Millnotes Mythril Edition++

Physical Constants

Speed of Light: $c = 2.998\times10^{8}\, m\cdot s^{-1}$

Avogadro's Constant: $N_A = 6.022\times10^{23}\, mole^{-1}$

Gravitation Constant: $G = 6.67\times10^{-11} m^3\cdot kg^{-1}\cdot s^{-2}$

Stefan-Boltzmann: $\sigma = 5.67\times10^{-8} W\cdot m^{-2}\cdot K^{-4}$

Permittivity of Vacuum: $\varepsilon_0 = 8.854\times10^{-12} F\cdot m^{-1}$

Permeability of Vacuum: $\mu_0 = 4\pi\times10^{-7} H\cdot m^{-1}$

Boltzmann Constant: $k_b = 8.617\times10^{-5} eV\cdot K^{-1}$

Fine Structure Constant: $\alpha = 7.297\times10^{-3}$

Electronic Charge: $e = 1.602\times10^{-19}\, C$

Planks Constant: $h = 4.136\times10^{-15}\, eV\cdot s$

Proton Mass: $m_p = 1.673\times10^{-27}\, kg$

Neutron Mass: $m_n = 1.675\times10^{-27}\, kg$

Electron Mass: $m_e = 9.11\times10^{-31}\, kg$

Rydberg Constant: $R_\infty = 1.097\times10^{7}\, m^{-1}$

Bohr Magneton: $\mu_B = 9.274\times10^{-24}\, J\cdot T^{-1}$

Bohr Radius: $a_0 = 5.291\times10^{-11}\, m$

$1eV = 1.602\times10^{-19} J$ $\quad 1T = 1\times10^{4}\mathrm{G}$ $\quad 1ChrisMiller = 1.81\mathrm{m}$ $\quad 1\mathrm{barn} = 1\times10^{-24} cm^2$ $\quad 1cal = 4.186J$ $\quad 1torr = 133.3Pa$

$1u = 1.66\times10^{-27} kg = 931.49\, {}^{MeV}\!/_{c^2}$ $\quad 1atm = 1.013\times10^{5}\mathrm{Pa}$ $\quad 1\mathrm{N} = 1\times10^{5}\,\mathrm{dyne}$ $\quad 1A = 1\times10^{-10} m$ $\quad 1J = 1\times10^{7} ergs$

Derivatives

$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$ $\qquad \frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ $\qquad \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$\frac{d}{dx}e^u = e^u\frac{du}{dx}$ $\qquad \frac{d}{dx}\log_a u = \frac{1}{u\ln(a)}\frac{du}{dx}$ $\qquad \frac{d}{dx}a^u = a^u\ln(a)\frac{du}{dx}$

$\frac{d}{dx}\sin(u) = \cos(u)\frac{du}{dx}$ $\qquad \frac{d}{dx}\cos(u) = -\sin(u)\frac{du}{dx}$ $\qquad \frac{d}{dx}\tan(u) = \sec^2(u)\frac{du}{dx}$

$\frac{d}{dx}\csc(u) = -\csc(u)\cot(u)\frac{du}{dx}$ $\qquad \frac{d}{dx}\sec(u) = \sec(u)\tan(u)\frac{du}{dx}$ $\qquad \frac{d}{dx}\cot(u) = -\csc^2(u)\frac{du}{dx}$

$\frac{d}{dx}\sin^{-1}(u) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$ $\qquad \frac{d}{dx}\cos^{-1}(u) = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$ $\qquad \frac{d}{dx}\tan^{-1}(u) = \frac{1}{1+u^2}\frac{du}{dx}$

$\frac{d}{dx}\csc^{-1}(u) = \frac{-1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$ $\qquad \frac{d}{dx}\sec^{-1}(u) = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$ $\qquad \frac{d}{dx}\cot^{-1}(u) = \frac{-1}{1+u^2}\frac{du}{dx}$

Trigonometric Identities

$\sin^2(\theta) + \cos^2(\theta) = 1$ $\qquad e^{i\theta} = \cos(\theta) + i\sin(\theta)$

$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$ $\qquad \csc\theta = \frac{1}{\sin\theta}$

$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$ $\qquad \sec\theta = \frac{1}{\cos\theta}$

$\sin(3\theta) = 3\sin(\theta) - 4\sin^3(\theta)$ $\qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cos(3\theta) = 4\cos^3(\theta) - 3\cos(\theta)$

$\cos^2(\theta) = \frac{1+\cos(2\theta)}{2}$ $\qquad \sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$

$\sin(A)\sin(B) = \frac{1}{2}\{\cos(A-B) - \cos(A+B)\}$

$\sin(A)\cos(B) = \frac{1}{2}\{\sin(A-B) + \sin(A+B)\}$

$\cos(A)\cos(B) = \frac{1}{2}\{\cos(A-B) + \cos(A+B)\}$

Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos(C)$

$\cos(\theta) = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$ $\qquad \sin(\theta) = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$

$\sin(i\theta) = i\sinh(\theta)$ $\qquad \cos(i\theta) = \cosh(\theta)$

$\sin(A\pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$

$\cos(A\pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$

$\tan(A\pm B) = \frac{\tan(A)\pm\tan(B)}{1\mp\tan(A)\tan(B)}$

$\sin(A) + \sin(B) = 2\sin(\frac{A+B}{2})\cos(\frac{A-B}{2})$

$\sin(A) - \sin(B) = 2\cos(\frac{A+B}{2})\sin(\frac{A-B}{2})$

$\cos(A) + \cos(B) = 2\cos(\frac{A+B}{2})\cos(\frac{A-B}{2})$

$\cos(A) - \cos(B) = -2\sin(\frac{A+B}{2})\sin(\frac{A-B}{2})$

Law of Sines: $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$

Gaussian Integrals

$\int_0^\infty x^n e^{-ax}dx = \frac{n!}{a^{n+1}}$ $\qquad \int_0^\infty x^{2n} e^{\frac{-x^2}{a^2}}dx = \sqrt{\pi}\frac{(2n)!}{n!}\left(\frac{a}{2}\right)^{2n+1}$

$\int_0^\infty x^{2n+1} e^{\frac{-x^2}{a^2}}dx = \frac{n!}{2}a^{2n+2}$ $\qquad \int_{-\infty}^\infty e^{-(ax^2+bx+c)}dx = \sqrt{\frac{\pi}{a}}e^{\frac{b^2-4ac}{4a}}$

Linear

$\vec{A}\cdot\vec{B} = |A||B|\cos(\theta) = A_xB_x + A_yB_y + A_zB_z$ $\qquad \|A\| = \sqrt{\vec{A}\cdot\vec{A}} = \sqrt{a_x^2 + a_y^2 + a_z^2}$

$$\vec{A}\times\vec{B} = |A||B|\sin(\theta)\hat{n} = \begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix} = \hat{x}\begin{vmatrix}a_y & a_z\\ b_y & b_z\end{vmatrix} - \hat{y}\begin{vmatrix}a_x & a_z\\ b_x & b_z\end{vmatrix} + \hat{z}\begin{vmatrix}a_x & a_y\\ b_x & b_y\end{vmatrix}$$

$\det(\boldsymbol{AB}) = \det(\boldsymbol{A})\det(\boldsymbol{B})$ $\qquad \det(\boldsymbol{A}^{-1}) = \frac{1}{\det(\boldsymbol{A})}$ $\qquad \det(c\boldsymbol{A}) = c^{rows}\det(\boldsymbol{A})$

$\boldsymbol{M} = \begin{pmatrix}m_{11} & m_{12}\\ m_{21} & m_{22}\end{pmatrix}$ $\qquad \det\boldsymbol{M} = m_{11}m_{22} - m_{12}m_{21}$ $\qquad \boldsymbol{M}^{-1} = \frac{1}{\det\boldsymbol{M}}\begin{pmatrix}m_{22} & -m_{12}\\ -m_{21} & m_{11}\end{pmatrix}$

$A\vec{x} = \lambda\vec{x} \to \det(\boldsymbol{A} - \lambda\boldsymbol{I}) = 0$ $\qquad proj_v u = \frac{\vec{u}\cdot\vec{v}}{\|v\|^2}\vec{v}$ $\qquad \frac{\partial f}{\partial u} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial u} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial u}$

$Symmetric: \boldsymbol{C} = \boldsymbol{C}^T$ $\qquad Orthogonal: \boldsymbol{C}^T = \boldsymbol{C}^{-1}$ $\qquad Unitary: \boldsymbol{C}^\dagger = \boldsymbol{C}^{-1}$

$Adjoint: \boldsymbol{C}^\dagger = \bar{\boldsymbol{C}}^T$ $\qquad Hermitian: \boldsymbol{C}^\dagger = \boldsymbol{C}$ $\qquad Normal: [\boldsymbol{C}^\dagger, \boldsymbol{C}] = 0$

$Divergence: \int(\nabla\cdot\boldsymbol{A})d\tau = \oint\boldsymbol{A}\cdot d\boldsymbol{a}$ $\qquad Stokes: \int(\nabla\times\boldsymbol{A})\cdot d\boldsymbol{a} = \oint\boldsymbol{A}\cdot d\boldsymbol{l}$

Taylor Series

$T(x) = \sum_{n=0}^\infty \frac{f^{(n)}(a)}{n!}(x-a)^n$

$e^x = \sum_{n=0}^\infty \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!}\ldots$

$\frac{1}{1-x} = \sum_{n=0}^\infty x^n = 1 + x + x^2\ldots$

$\frac{1}{1+x} = \sum_{n=0}^\infty (-1)^n x^n = 1 - x + x^2\ldots$

$(a+x)^n = \sum_{k=0}^n \binom{n}{k}a^{n-k}x^k = a^n + na^{n-1}x\ldots$

$\sin(x) = \sum_{n=0}^\infty (-1)^n\frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!}\ldots$

$\cos(x) = \sum_{n=0}^\infty (-1)^n\frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}\ldots$

$\ln(1+x) = \sum_{n=1}^\infty (-1)^{n-1}\frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3}\ldots$

$\tan^{-1}(x) = \sum_{n=0}^\infty (-1)^n\frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5}\ldots$

Complex Analysis

$z = x + iy = re^{i\theta}$ $\qquad \bar{z} = x - iy = re^{-i\theta}$

$|z| = r = \sqrt{x^2 + y^2}$ $\qquad \theta = \arg(z) = \tan^{-1}\left(\frac{y}{x}\right)$

$z = re^{i\theta} = r(\cos\theta + i\sin\theta)$ $\qquad |z_1\cdot z_2| = |z_1|\cdot|z_2|$

$\ln(z) = \ln(r) + i(\theta + 2\pi n)$ $\qquad z\cdot\bar{z} = |z|^2$

$\arg(z_1z_2) = \arg(z_1) + \arg(z_2)$ $\qquad \arg(\bar{z}) = -\arg(z)$

$$f^{(n)}(z_0) = \frac{n!}{2\pi i}\int_C \frac{f(z)}{(z-z_0)^{n+1}}dz$$

$$\int_C f(z)dz = 2\pi i\sum \text{enclosed residues}$$

$$a_{-1} = \frac{1}{(m-1)!}\frac{d^{m-1}}{dz^{m-1}}\left[(z-z_0)^m f(z)\right]_{z=z_0}$$

$$\underset{z=z_0}{Res}\frac{p(z)}{q(z)} = \frac{p(z_0)}{q'(z_0)}$$

Homework 11: *The Riemann Hypothesis*

1. Prove that the real part of any non-trivial zero of the Riemann zeta function is 1/2.
Due to the obviousness of the statement, the proof is left to the grader as an exercise. **QED**

Special Functions

Bessel Function

For small x:

$J_n(x) \sim \frac{1}{2^n n!}x^n$ $\qquad Y_0(x) \sim \frac{2}{\pi}\ln x$ $\qquad Y_0(x) \sim \frac{2^n(n-1)!}{\pi}x^{-n}$

For large x:

$J_n(x) \sim \sqrt{\frac{2}{\pi x}}\cos\left[x - \frac{\pi}{4}(2n+1)\right]$ $\qquad J_n(x) \sim \sqrt{\frac{2}{\pi x}}\sin\left[x - \frac{\pi}{4}(2n+1)\right]$

$j_0(x) = \frac{\sin x}{x}$ $\qquad \eta_0(x) = -\frac{\cos x}{x}$

$j_1(x) = \frac{\sin x}{x^2} - \frac{\cos x}{x}$ $\qquad \eta_1(x) = -\frac{\cos x}{x^2} - \frac{\sin x}{x}$

$j_l(x) \to \left(\frac{2^l l!}{(2l+1)!}\right)x^l \quad x \ll 1$ $\qquad \eta_l(x) \to -\frac{(2l!)}{2^l l!}\frac{1}{x^{l+1}} \quad x \ll 1$

Legendre

$P_0 = 1$ $\qquad P_3 = \frac{1}{2}(5x^2 - 3x)$

$P_1 = x$ $\qquad P_4 = \frac{1}{8}(34x^4 - 30x^2 + 3)$

$P_2 = \frac{1}{2}(3x^2 - 1)$ $\qquad P_5 = \frac{1}{8}(63x^5 - 70x^3 + 15x)$

Associated Legendre

$P_l^{-m}(x) = (-1)^m\frac{(l-m)!}{(l+m)!}P_l^m(x)$

$P_0^0 = 1$ $\qquad P_2^2 = 3\sin^2\theta$

$P_1^1 = \sin\theta$ $\qquad P_2^1 = 3\sin\theta\cos\theta$

$P_1^0 = \cos\theta$ $\qquad P_2^0 = \frac{1}{2}(3\cos^2\theta - 1)$

Spherical Harmonics

$Y_0^0 = \sqrt{\frac{1}{4\pi}}$ $\qquad Y_2^{\pm2} = \mp\sqrt{\frac{15}{32\pi}}\sin^2\theta e^{\pm2i\phi}$

$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$ $\qquad Y_3^0 = \sqrt{\frac{7}{16\pi}}(5\cos^3\theta - 3\cos\theta)$

$Y_1^{\pm1} = \mp\sqrt{\frac{3}{8\pi}}\sin\theta e^{\pm i\phi}$ $\qquad Y_3^{\pm1} = \mp\sqrt{\frac{21}{64\pi}}\sin\theta(5\cos^2\theta - 1)e^{\pm i\phi}$

$Y_2^0 = \mp\sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1)$ $\qquad Y_3^{\pm2} = \mp\sqrt{\frac{105}{32\pi}}\sin^2\theta\cos\theta e^{\pm2i\phi}$

$Y_2^{\pm1} = \mp\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{\pm i\phi}$ $\qquad Y_3^{\pm3} = \mp\sqrt{\frac{35}{64\pi}}\sin^3\theta e^{\pm3i\phi}$

Hankel

$H_0 = 1$ $\qquad H_3 = 8x^3 - 12x$

$H_1 = 2x$ $\qquad H_4 = 16x^4 - 48x^2 + 12$

$H_2 = 4x^2 - 2$ $\qquad H_5 = 32x^5 - 160x^3 + 120x$

How to physically prove: a→b

1. Assume a
2. Give a plausible argument, preferably using an example
3. QED

Fourier Series

(period=2L)

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^\infty a_n\cos\left(\frac{n\pi x}{L}\right) + b_n\sin\left(\frac{n\pi x}{L}\right)$$

$$a_n = \frac{1}{L}\int_{-L}^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx \qquad b_n = \frac{1}{L}\int_{-L}^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$f(x) = \sum_{n=-\infty}^\infty c_n e^{\frac{in\pi x}{L}} \qquad c_n = \frac{1}{2L}\int_{-L}^L f(x)e^{\frac{-in\pi x}{L}}dx$$

Limits

$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\frac{d}{dx}f(x)}{\frac{d}{dx}g(x)}$ $\qquad \lim_{n\to\infty}\frac{x^n}{n!} = 0$

$\lim_{n\to\infty}\left(1 + \frac{x}{n}\right)^n = e^x$ $\qquad \lim_{x\to0}\frac{\sin x}{x} = 1$

$\sin\left([2n-1]\frac{\pi}{2}\right) = (-1)^n$ $\qquad \cos(n\pi) = (-1)^n$

Pot Luck

$$\sum_{i=1}^{n} i = \frac{n}{2}(n+1)$$

$$\sum_{i=1}^{n} i^2 = \frac{n}{6}(n+1)(2n+1)$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2}{4}(n+1)^2$$

$$\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt$$

$$n! = \Gamma(n+1) = n\Gamma(n)$$

$$n! \simeq n^{n+\frac{1}{2}}e^{-n}\sqrt{2\pi}$$

$$\ln(ax) = \ln(a) + \ln(x) \qquad \ln x^n = n\ln x$$

$$\ln\left(\frac{a}{x}\right) = \ln(a) - \ln(x) \qquad \log_a x = \frac{\ln(x)}{\ln(a)}$$

$$\sum_{n=0}^{n-1} ar^n = \frac{1-r^n}{1-r} \qquad l = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$[A,B] = AB - BA = -[B,A]$$

$$[A,B]^\dagger = [B^\dagger, A^\dagger]$$

$$[A+B,C] = [A,C] + [B,C]$$

$$[AB,C] = A[B,C] + [A,C]B$$

$$[A,[B,C]] = [B,[A,C]] - [C,[A,B]]$$

The Miracle Operator: ☺

☺(Wrong Answer) = Right Answer

$$\int_a^b \delta(x)dx = \begin{cases} 1 & \text{if } a<0<b \\ 0 & \text{otherwise} \end{cases} \qquad \delta(x) \simeq \frac{n}{\sqrt{\pi}}e^{-n^2x^2}$$

$$\int_a^b f(x)\delta(x-x_0)dx = f(x_0) \qquad \delta(ax) = |a|^{-1}\delta(x)$$

$$\nabla r = \frac{\vec{r}}{r} \qquad \nabla\left(\frac{1}{r}\right) = \frac{-\vec{r}}{r^3}$$

$$\nabla r^2 = 2\vec{r} \qquad \nabla\left(\frac{1}{r^2}\right) = \frac{-2\vec{r}}{r^4}$$

$$\nabla\cdot\vec{r} = 3 \qquad \nabla\cdot\left(\frac{\vec{r}}{r^2}\right) = \frac{1}{r^2}$$

$$\nabla\cdot(r\vec{r}) = 4r \qquad \nabla\cdot\left(\frac{\vec{r}}{r^3}\right) = 4\pi\delta(\vec{r})$$

$$a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + \cdots + ab^{n-2} + b^{n-1}) \quad (n>1)$$

Rectangular

$$d\vec{l} = dx\hat{x} + dy\hat{y} + dz\hat{z}$$

$$\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

Probability

$$\langle x\rangle = \frac{1}{N}\sum_{i=1}^{N} x_i \qquad \sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \langle x\rangle)^2} \qquad \sigma_m = \frac{\sigma}{\sqrt{N}}$$

$$(\sigma R)^2 = \left(\sigma x\frac{\partial R}{\partial x}\right)^2 + \left(\sigma y\frac{\partial R}{\partial y}\right)^2 + \left(\sigma z\frac{\partial R}{\partial z}\right)^2 \ldots$$

$$\chi^2 = \sum_{i=1}^{N}\left(\frac{y_i - f(x_i)}{\sigma_i}\right)^2 \qquad \chi^2_{reduced} = \frac{\chi^2}{N - N_{fit\ parameters}}$$

$$_nP_r = \frac{n!}{(n-r)!} \qquad _nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$\langle x\rangle = \int_{-\infty}^{\infty} xf(x)dx \qquad \sigma^2 = \langle x^2\rangle - \langle x\rangle^2$$

Geometry

A = Area P = Perimeter S = Surface Area V = Volume

r = Radius h = Height a/b = Semi-Axes

Cylinder: $V = \pi r^2 h \quad S = 2\pi r(h+r)$ Sphere: $V = \frac{4}{3}\pi r^3 \quad S = 4\pi r^2$

Cone: $V = \frac{1}{3}\pi r^2 h \quad S_{lat} = \pi r\sqrt{r^2+h^2}$ Ellipse: $A = \pi ab \quad P \approx 2\pi\sqrt{\frac{a^2+b^2}{2}}$

Torus: $V = \frac{\pi^2}{4}(r_2^2 - r_1^2)(r_2 - r_1) \quad S = \pi^2(r_1+r_2)(r_2-r_1)$

Vector Identities

$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A} \qquad \vec{A}\times\vec{B} = -\vec{B}\times\vec{A} \qquad \vec{A}\cdot(\vec{B}+\vec{C}) = (\vec{A}\cdot\vec{B}) + (\vec{A}\cdot\vec{C})$$

$$\nabla\cdot(f\vec{A}) = f(\nabla\cdot\vec{A}) + \vec{A}\cdot\nabla f \qquad \vec{A}\times(\vec{B}+\vec{C}) = (\vec{A}\times\vec{B}) + (\vec{A}\times\vec{C})$$

$$\nabla\times(f\vec{A}) = f(\nabla\times\vec{A}) + (\nabla f)\times\vec{A} \qquad \nabla\cdot(\nabla\times\vec{A}) = 0$$

$$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B}) \qquad \nabla\times(\nabla f) = 0$$

$$(\vec{A}\times\vec{B})\times\vec{C} = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{A}(\vec{B}\cdot\vec{C}) \qquad \nabla\cdot(\nabla f) = \nabla^2 f$$

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B}) \qquad \nabla(fg) = g\nabla f + f\nabla g$$

$$\nabla\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\nabla\times\vec{A}) - \vec{A}\cdot(\nabla\times\vec{B}) \qquad \nabla\times(\nabla\times\vec{A}) = \nabla(\nabla\cdot\vec{A}) - \nabla^2\vec{A}$$

$$(\vec{A}\times\vec{B})\cdot(\vec{C}\times\vec{D}) = (\vec{A}\cdot\vec{C})(\vec{B}\cdot\vec{D}) - (\vec{A}\cdot\vec{D})(\vec{B}\cdot\vec{C}) \qquad \pi = 3 \qquad 12 \sim \infty$$

Fourier Transforms

$$F(s) = \int_{-\infty}^{\infty} f(x)e^{-2\pi ixs}dx \qquad f(x) = \int_{-\infty}^{\infty} F(s)e^{2\pi ixs}ds$$

$$f(x) * g(x) = \int_{-\infty}^{\infty} f(u)g(x-u)du \qquad \frac{d}{dx}[f(x)*g(x)] = \frac{df(x)}{dx}*g(x) = \frac{dg(x)}{dx}*f(x)$$

$$\frac{d^n}{dx^n}f(x) \rightleftharpoons (2\pi is)^n F(s) \qquad f(ax) \rightleftharpoons \frac{1}{|a|}F\left(\tfrac{s}{a}\right) \qquad f(x-a) \rightleftharpoons e^{-2\pi ias}F(s)$$

Physics

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}_k} - \frac{\partial L}{\partial x_k} + \sum_{a=1}^{m}\lambda_a\frac{\partial f_a}{\partial x_k} = 0$$

$$\nabla\cdot\vec{D} = \rho_f \qquad \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad \nabla\cdot\vec{B} = 0 \qquad \nabla\times\vec{H} = \vec{J}_f + \frac{\partial\vec{D}}{\partial t}$$

$$\vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right) \qquad \vec{E} = -\nabla V - \frac{\partial\vec{A}}{\partial t} \qquad \vec{B} = \nabla\times\vec{A}$$

$$V_{dip} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^2} \qquad \vec{p} = \int\vec{r}\rho(\vec{r})d\tau \qquad \vec{D} = \varepsilon_0\vec{E} + \vec{P} \qquad \vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$$

$$U = \frac{1}{2}\int\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2 d\tau \qquad \vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} \qquad P = \frac{\mu_0}{6\pi c}q^2a^2$$

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi \qquad H = -\frac{\hbar^2}{2m}\nabla^2 + V \qquad \mathrm{p} = -i\hbar\nabla$$

$$\frac{d\langle Q\rangle}{dt} = \frac{i}{\hbar}\langle[H,Q]\rangle + \left\langle\frac{dQ}{dt}\right\rangle \qquad \sigma_A\sigma_B \geq \frac{1}{2i}\langle[A,B]\rangle$$

$$S_\pm = S_x + iS_y \qquad S^2 = S_\pm S_\mp + S_z^{\,2} \mp \hbar S_z$$

$$\left[S_x, S_y\right] = i\hbar S_z \qquad \vec{S}_1\cdot\vec{S}_2 = S_{1z}S_{2z} + \frac{1}{2}(S_{1+}S_{2-} + S_{1-}S_{2+})$$

$$\psi_n^1 = \sum_{m\neq n}\frac{\langle\psi_m^0|H'|\psi_n^0\rangle}{E_n^0 - E_m^0}\psi_m^0 \qquad E_n^2 = \sum_{m\neq n}\frac{\langle\psi_m^0|H'|\psi_n^0\rangle^2}{E_n^0 - E_m^0}$$

$$c = \frac{i}{\hbar}\int H'_{ba}e^{i\frac{E_b - E_a}{\hbar}t}dt \qquad a_\pm = \frac{1}{\sqrt{2\hbar m\omega}}(\mp ip + m\omega x)$$

$$Z = \sum_s e^{-\beta E_s} \qquad Q = \int e^{-\beta H}dp^3pdr^3 \qquad dU = \tau d\sigma - pdV + \mu dN$$

$$\langle U\rangle = -\frac{\partial\ln Z}{\partial\beta} \qquad C_v = \frac{\partial\langle U\rangle}{\partial T} \qquad f(\varepsilon) = \frac{1}{e^{(\varepsilon-u)/t}\pm 1}$$

Cylindrical

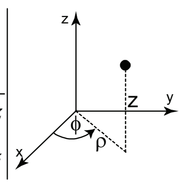
$$d\vec{l} = d\rho\hat{\rho} + \rho d\phi\hat{\phi} + dz\hat{z} \qquad dV = \rho d\rho d\phi dz$$

$$\nabla f = \frac{\partial f}{\partial\rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial f}{\partial\phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z}$$

$$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial f}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial\phi^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\nabla\cdot\vec{A} = \frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\vec{A} = \begin{vmatrix} \frac{\hat{\rho}}{\rho} & \hat{\phi} & \frac{\hat{z}}{\rho} \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$



$$x = \rho\cos(\phi) \qquad \hat{x} = \cos(\phi)\hat{\rho} - \sin(\phi)\hat{\phi}$$

$$y = \rho\sin(\phi) \qquad \hat{y} = \sin(\phi)\hat{\rho} + \cos(\phi)\hat{\phi}$$

$$z = z \qquad \hat{z} = \hat{z}$$

$$\rho = \sqrt{x^2 + y^2} \qquad \hat{\rho} = \cos(\phi)\hat{x} + \sin(\phi)\hat{y}$$

$$\phi = \tan^{-1}(y/x) \qquad \hat{\phi} = -\sin(\phi)\hat{x} + \cos(\phi)\hat{y}$$

Laplace Transforms

$$F(s) = \int_0^\infty f(t)e^{-st}dt \qquad (f*g)(t) = \int_0^t f(t-s)g(s)ds$$

$f(t)$	$F(s)$		
t^n	$\frac{n!}{s^{n+1}}$	$f'(t)$	$sF(s) - f(0)$
		$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
e^{at}	$\frac{1}{s-a}$	$e^{at}\cos(kt)$	$\frac{s-a}{(s-a)^2+k^2}$
$\sin(kt)$	$\frac{k}{s^2+k^2}$	$e^{at}\sin(kt)$	$\frac{k}{(s-a)^2+k^2}$
$\cos(kt)$	$\frac{s}{s^2+k^2}$	$t^nf(t)$	$(-1)^nF^{(n)}(s)$
$e^{at}f(t)$	$F(s-a)$		
$\delta(t-a)$	e^{-as}	$J_n(at)$	$\frac{\left(\sqrt{s^2+a^2}-s\right)^n}{a^n\sqrt{s^2+a^2}}$
t^ne^{at}	$\frac{n!}{(s-a)^{n+1}}$		

Rotations

$$R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} \qquad R_y(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$R_z(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{Euler} = R_z(-\gamma)R_y(-\beta)R_z(-\alpha)$$

Extrema

Stationary Point if: $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0$ at (x_0, y_0)

Maximum: $\frac{\partial^2 f}{\partial x^2} < 0$ and $\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2} > \left(\frac{\partial^2 f}{\partial x\partial y}\right)^2$

Minimum: $\frac{\partial^2 f}{\partial x^2} > 0$ and $\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2} > \left(\frac{\partial^2 f}{\partial x\partial y}\right)^2$

Saddle Point: $\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2} < \left(\frac{\partial^2 f}{\partial x\partial y}\right)^2$

Spherical

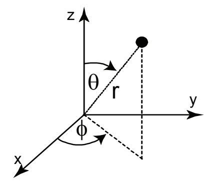
$$d\vec{l} = dr\hat{r} + rd\theta\hat{\theta} + r\sin(\theta)d\phi\hat{\phi} \qquad dV = r^2\sin(\theta)drd\theta d\phi$$

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial\theta}\hat{\theta} + \frac{1}{r\sin(\theta)}\frac{\partial f}{\partial\phi}\hat{\phi}$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin(\theta)}\frac{\partial}{\partial\theta}\left(\sin(\theta)\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2(\theta)}\frac{\partial^2 f}{\partial\phi^2}$$

$$\nabla\cdot\vec{A} = \frac{1}{r^2}\frac{\partial(r^2A_r)}{\partial r} + \frac{1}{r\sin(\theta)}\frac{\partial(A_\theta\sin(\theta))}{\partial\theta} + \frac{1}{r\sin(\theta)}\frac{\partial A_\phi}{\partial\phi}$$

$$\nabla\times\vec{A} = \begin{vmatrix} \frac{\hat{r}}{r^2\sin(\theta)} & \frac{\hat{\theta}}{r\sin(\theta)} & \frac{\hat{\phi}}{r} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ A_r & rA_\theta & rA_\phi\sin(\theta) \end{vmatrix}$$



$$x = r\sin(\theta)\cos(\phi) \qquad \hat{x} = \sin(\theta)\cos(\phi)\hat{r} + \cos(\theta)\cos(\phi)\hat{\theta} - \sin(\phi)\hat{\phi}$$

$$y = r\sin(\theta)\sin(\phi) \qquad \hat{y} = \sin(\theta)\sin(\phi)\hat{r} + \cos(\theta)\sin(\phi)\hat{\theta} + \cos(\phi)\hat{\phi}$$

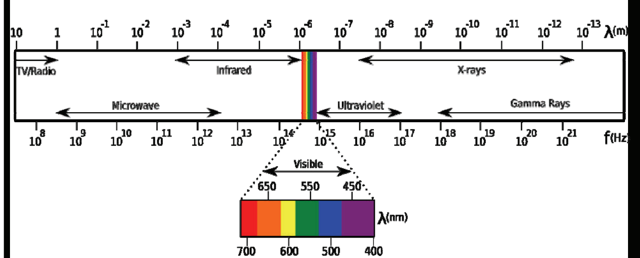
$$z = r\cos(\theta) \qquad \hat{z} = \cos(\theta)\hat{r} - \sin(\theta)\hat{\theta}$$

$$r = \sqrt{x^2+y^2+z^2} \qquad \hat{r} = \sin(\theta)\cos(\phi)\hat{x} + \sin(\theta)\sin(\phi)\hat{y} + \cos(\phi)\hat{z}$$

$$\theta = \tan^{-1}(\sqrt{x^2+y^2}/z) \qquad \hat{\theta} = \cos(\theta)\cos(\phi)\hat{x} + \cos(\theta)\sin(\phi)\hat{y} - \sin(\theta)\hat{z}$$

$$\phi = \tan^{-1}(y/x) \qquad \hat{\phi} = -\sin(\phi)\hat{x} + \cos(\phi)\hat{y}$$

Electromagnetic Spectrum



Periodic Table

1 H 1.01																	2 He 4.00
3 Li 6.94	4 Be 9.01											5 B 10.81	6 C 12.01	7 N 14.01	8 O 16.00	9 F 19.00	10 Ne 20.18
11 Na 22.99	12 Mg 24.31											13 Al 26.98	14 Si 28.09	15 P 30.97	16 S 32.06	17 Cl 35.45	18 Ar 39.95
19 K 39.10	20 Ca 40.08	21 Sc 44.96	22 Ti 47.88	23 V 50.94	24 Cr 52.00	25 Mn 54.94	26 Fe 55.85	27 Co 58.93	28 Ni 58.69	29 Cu 63.55	30 Zn 65.38	31 Ga 69.72	32 Ge 72.59	33 As 74.92	34 Se 78.96	35 Br 79.90	36 Kr 83.80
37 Rb 85.47	38 Sr 87.62	39 Y 88.91	40 Zr 91.22	41 Nb 92.91	42 Mo 95.94	43 Tc 98.1	44 Ru 101.1	45 Rh 102.9	46 Pd 106.4	47 Ag 107.9	48 Cd 112.4	49 In 114.8	50 Sn 118.7	51 Sb 121.8	52 Te 127.6	53 I 126.9	54 Xe 131.3
55 Cs 132.9	56 Ba 137.3	57 La 138.9	72 Hf 178.5	73 Ta 180.9	74 W 183.9	75 Re 186.2	76 Os 190.2	77 Ir 192.2	78 Pt 195.1	79 Au 197.0	80 Hg 200.6	81 Tl 204.4	82 Pb 207.2	83 Bi 209.0	84 Po (209)	85 At (210)	86 Rn (222)
87 Fr 223	88 Ra 226	89 Ac (227)	104 Rf (261)	105 Db (262)	106 Sg (263)	107 Bh (262)	108 Hs 186.2	109 Mt (268)	110 Ds (269)	777 Cm 181							
		58 Ce 140.1	59 Pr 140.9	60 Nd 144.2	61 Pm (145)	62 Sm 150.4	63 Eu 152.0	64 Gd 157.3	65 Tb 158.9	66 Dy 162.5	67 Ho 164.9	68 Er 167.3	69 Tm 168.9	70 Yb 173.0	71 Lu 175.0		
		90 Th 232.0	91 Pa (231)	92 U 238.0	93 Np (237)	94 Pu (244)	95 Am (243)	96 Cm (247)	97 Bk (247)	98 Cf (251)	99 Es (252)	100 Fm (257)	101 Md (258)	102 No (259)	103 Lr (260)